2012 Soil Mechanics I and Exercises Final Examination
2013/1/22 (Tue) 13:00 - 15:00 Kyotsu 155 • Kyotsu 1 • Kyotsu 3 • W2 Lecture room

Attention:
- There are four questions and four answer sheets. Write down your name and ID number on every answer sheet. Use one answer sheet for one question and answer in sequence from Question 1. If the front side of the answer sheet is not sufficient for answering that question, mention that and use the back side of that answer sheet.
- Carrying any personal belongings is prohibited. For any mischievous act, you will not be entitled to get the credit of this subject.
- Wherever necessary, show the units in your answers.

【Question 1】
In order to restore a river embankment damaged by an earthquake, a soil of 50,000 m³ estimated after compaction was needed. From the laboratory tests of soil from the excavation site, the water content was \( w = 15\% \), and the void ratio was \( e = 0.60 \), and the specific gravity of soil was \( G_s = 2.70 \). The soil was carried by truck to the construction site. The load capacity of the truck is 6.5 t.

The construction of embankment was done by compacting after adding water so that the water content becomes 20%. The dry density of soil after compaction was 1.80 t/m³.

Answer the following questions. The density of water is 1.0 t/m³.

(1) Find the saturation \( S_r \), the bulk density, and the dry density of soil at the excavation site.
(2) Find the total number of trucks needed for the construction.
(3) Find the total volume of soil excavated at the excavation site.
(4) Find the total volume of water added to obtain the prescribed water content. Assume that evaporation of water is negligible.
(5) The embankment is needed to be constructed under the best condition of soil so that the embankment fulfills its function. Explain what kind of test is needed for that, that is, the purpose and the procedure of the test.

【Question 2】Answer the following questions on the water flow in saturated soils. Assume that the water flow in soil is governed by Darcy’s law.

(1) Soil 1 with a cross-sectional area of \( 1.0 \times 10^{-2} \) m² is set up in the permeameter in which the specimen is supported by a mesh at the bottom. A constant-head difference across the specimen is maintained as shown in Figure 1. After a constant flow rate is established, the flow rate reaches \( 2.4 \times 10^{-5} \) m³/min. Determine the hydraulic conductivity (coefficient of permeability) of Soil 1. Assume that the specimen is fully saturated with water once the constant flow rate is established.
(2) Calculate the flow rate when Soil 1 is overlain by Soil 2 with a thickness of 0.1 m and a hydraulic conductivity of $1.0 \times 10^{-5}$ m/s as shown in Figure 2. Assume that Soils 1 and 2 are fully saturated with water, and a constant flow rate is established.

(3) Plot the variations of total, potential, and pressure heads in Soils 1 and 2 along the horizontal axis of a graph, with elevation as the vertical axis, under the constant-flow condition described in Question (2). Assume that the bottom level of Soil 1 is the datum.

**[Question 3]** Answer the following questions with reference to Fig. 3, where a normally consolidated clay layer lays between two layers of sand.

(1) When a uniform load of $P_0 = 40$ kN/m$^2$ is applied on the ground surface, find the total stress, the effective stress and the pore water pressure at the center of the clay layer (i.e., at a depth of 5 m) for the following three cases: i) before the application of the load, ii) just after the loading; and after sufficient time has elapsed for the settlement to finish. Note, the unit weight of water is 9.81 kN/m$^3$.

(2) Find the final settlement of the clay layer. Use the stress at the center (i.e., at a depth of 5 m) as a representative value of the stress of the full layer. Note, for the clay layer, the initial void ratio $e_0 = 2.0$ and the compression index $C_c = 2.0$. 

![Figure 1](image1.png)

![Figure 2](image2.png)
Use the following approximate common logarithm values:

\[
\begin{align*}
\log 1.8 &= 0.26, \\
\log 2.0 &= 0.30, \\
\log 2.2 &= 0.34, \\
\log 2.4 &= 0.38, \\
\log 2.6 &= 0.41, \\
\log 2.8 &= 0.45, \\
\log 3.0 &= 0.48
\end{align*}
\]

(3) Find the days needed for the consolidation to reach \( U = 80\% \). Use the following table of the relation between the degree of consolidation \( U \) and the time factor \( T_v \). [Take \( c_v = 1.5 \times 10^{-3} \text{ cm}^2/\text{s} \).]

<table>
<thead>
<tr>
<th>( U (%) )</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_v )</td>
<td>0.008</td>
<td>0.031</td>
<td>0.071</td>
<td>0.126</td>
<td>0.197</td>
<td>0.287</td>
<td>0.403</td>
<td>0.567</td>
<td>0.848</td>
</tr>
</tbody>
</table>

![Figure 3](image_url)
【Question 4】Answer the following questions related to consolidation of soils.

(1) Consider a stress state for soil element shown in Fig. 4. Calculate the shear stress and the vertical stress acting on the A-A’ plane (upward) which is inclined 45°. Draw a Mohr’s stress circle and indicate the pole. The stress components are $\sigma_{xx} = 50$ kPa, $\sigma_{yy} = 100$ kPa, $\tau = 20$ kPa.

(2) Describe Mohr-Coulomb’s failure criterion in terms of principal stresses and explain the criterion by a Mohr’s stress circle and its feature.
An embankment is constructed from the soil excavated from an excavation site. After completion, the total volume of this embankment is 20,000 m$^3$. The soil from the excavation site is investigated in its natural conditions and the following results are obtained:

- Bulk density: 1.80 t/m$^3$
- Water content: 16.0%
- Specific gravity of soil grains: 2.70

In addition, from the results of compaction tests conducted for the excavated soil, the optimum water content is found to be equal to 19.0%, at which the maximum dry density is equal to 1.80 t/m$^3$. During the construction of the embankment, water is sprayed and the optimum water content condition is reached. At this optimum water content condition, the maximum dry density of the embankment is 90.0% of the maximum dry density obtained during the compaction test (i.e., degree of compaction = 90.0%). Answer the following questions considering the density of water as 1.00 t/m$^3$:

1. Calculate the void ratio, porosity, degree of saturation and dry density of the soil at the excavation site in the natural condition.
2. Calculate the volume and mass of the soil to be excavated from the excavation site in the natural condition.
3. Calculate the mass of water that is necessary to be sprayed on 1 m$^3$ of soil which is in natural condition at the excavation site.
4. Calculate the degree of saturation of the embankment after the completion of compaction.
The following figures represent three stages of an experiment on one-dimensional flow of water in the vertical direction through the soil contained in a percolation cylinder, satisfying Darcy’s Law. The cross-sectional area of the cylinder is 1000 cm$^2$, the saturated unit weight of the soil specimen is $\gamma_{sat} = 19.0$ kN/m$^3$, and the unit weight of water is $\gamma_w = 9.81$ kN/m$^3$. Assuming that the datum is located at the bottom of the soil specimen, answer the following questions:

1. When a constant head permeability test was carried out under the conditions shown in figure (a), the flow of water through the soil specimen was 120 cm$^3$/min. Calculate the coefficient of permeability of the soil. In addition, calculate the total head, the elevation head, the pressure head, and the effective stress at the bottom of the soil specimen.

2. Next, as shown in figure (b), the water table at the bottom of the soil specimen was raised to a height of 4 m. Calculate the total head, the elevation head, the pressure head, and the effective stress at the bottom of the soil specimen under this new condition.

3. Later, as shown in figure (c), the height of the soil specimen was lowered, and a constant head permeability test was carried out. Other than the height of the soil specimen, the experimental conditions are identical as those in (2). The height of the soil specimen was gradually lowered from its original 2 m, until quicksand (boiling) conditions were reached at a height of $L$ (m). Calculate the value of $L$.

4. Describe the falling head permeability test, compare it to the constant head permeability test, and mention under which conditions each one is used.
The ground shown in figure 1 consists of a sand and a normally consolidated clay layers on top of an impermeable rock. The groundwater level coincides with the ground surface (i.e., the upper surface of the sand layer). To answer the following questions, assume that the unit weight of water is $\gamma_w = 9.8 \text{ kN/m}^3$. To calculate the settlement due to one-dimensional consolidation, consider that the physical properties of the middle of the clay layer correspond to the whole layer. Assume that sand is a full drainage material and, therefore, no excess pore water pressure occurs within the sand layer.

1. Calculate the initial total stress, pore water pressure (hydrostatic pressure), and effective stress, at the middle of the clay layer.

2. Assume that the groundwater level, which coincided with the ground surface at the beginning of this analysis, rises 1 m in a short period of time, as shown in figure 2. Calculate the effective stress at the middle of the clay layer under the new condition, after a sufficient amount of time passes, and also determine the settlement of the clay layer.

3. Instead of the groundwater level rise of (2), assume that it drops, from the initial condition described in (1), to the upper side of the clay layer, in a short period of time. Calculate the effective stress at the middle of the clay layer under this condition, after a sufficient amount of time passes, as well as the settlement of the clay layer. Since the water table is now located below the sand layer, the latter will be in an unsaturated condition with a unit weight of $\gamma_s = 16.2 \text{ kN/m}^3$. For this sand layer, assume that the changes in depth due to subsidence, as well as of its physical properties, are negligible.

4. Calculate the time required for the consolidation process under the conditions described in (3) to reach 90% (Time factor $T_v = 0.848$). If the impermeable layer at the bottom of the clay layer was a drainage boundary instead, how much time would it take for the consolidation process to reach 90%?
[4] Answer the following questions:

(1) Explain unconsolidated undrained shear strength (UU strength).

(2) Explain the behavior of loosely and densely compacted sands during consolidated drained shear test (CD test) using the graphs shown below. Here, \( \sigma_1 - \sigma_3 \) is deviator stress, \( \varepsilon \) is axial strain and \( \varepsilon_v \) is volumetric strain (take compression as positive).

(i) Loose sand

(ii) Dense sand

(3) A soil element is under the action of stresses shown below. At this condition, obtain the stress components acting on the horizontal plane A-A by drawing Mohr’s stress circle using the Pole method. Also, do show the pole in the figure drawn.
Attention:
- The exam consists of four questions for which you are provided with four answer sheets. Write down your name and ID number on every answer sheet. Use one answer sheet per question and answer them in sequence, starting from Question [1] If the space provided in the answer sheet is insufficient, use the back page after clearly mentioning so (for example, “continued on back page”).
- In addition to personal writing instruments, non-programmable calculators are permitted, but programmable calculators and all types of mobile phones are prohibited. Any attempts at cheating on the exam will result in failed credit of the course and serious penalties.
- Wherever necessary, specify the units in your answers.

[Question 1] In determining the causes and remedial treatments of landslides, it is important to understand the amount and state of soils at disaster sites. Herein, the ground displaced from its original position on slope as shown in Figure 1 becomes loosened as shown in Figure 2 by movement in landslide. Still, it is assumed that the displaced ground is homogeneous both before and after the failure. A field investigation based on topographic survey revealed that the original volume of the displaced ground was 2000 m$^3$ and it increased to 2500 m$^3$ after the failure. Soil samples were taken immediately after the landslide from the displaced ground and, after a laboratory test, the water content was measured to be $w = 20.0\%$. In addition, according to the results of a past soil survey, it is known that density of soil particle for the original ground is $\rho_s = 2.60\text{ g/cm}^3$ (or the specific gravity of soil particle $G_s = 2.60$) and dry density is $\rho_d = 1.60\text{ g/cm}^3$. Under these conditions, determine the following properties showing the calculation processes. Use gravitational acceleration $9.8\text{ m/s}^2$, and include units whenever necessary.

\[ V = 2\ 000\ m^3 \]
\[ V = 2\ 500\ m^3 \]

Figure 1 Before landslide

Figure 2 After landslide
(1) Void ratio of the original ground
(2) Total weight of the displaced ground
(3) Bulk density of the displaced ground
(4) Dry density of the displaced ground
(5) Degree of saturation of the sloping ground just before the occurrence of the landslide
(by assuming that the water content before and after the occurrence of landslide is unchanged)

[Question 2] For the following questions about flow of water in fully saturated soils, assume that Darcy’s Law applies.

(1) An undisturbed sample of soil was taken from the field to measure permeability in a laboratory setting. Assume that water density is $\rho_w = 1.0 \times 10^3$ kg/m$^3$, and acceleration of gravity is $g = 9.8$ m/s$^2$.

1) Before running the permeability test, the bulk density $\rho_b$, water content $w$, and soil particle density $\rho_s$ of the soil sample were measured, and the following values were obtained: $\rho_b = 2.0 \times 10^3$ kg/m$^3$, $w = 20\%$, $\rho_s = 2.70 \times 10^3$ kg/m$^3$. Calculate the void ratio $e$ of this specimen.

2) Find the saturated unit volume weight $\gamma_{sat}$ of this specimen.

3) Figure 3 shows a diagram of the constant head apparatus used to measure the permeability of this soil sample. The cylindrical mold containing the soil sample has

![Diagram of the apparatus used to run the constant head permeability test](image-url)
an inner diameter of 6.0×10^{-2} m and a height of 0.50 m. Plot the total head, elevation head, and water pressure diagrams for the whole range z = 0 m to z = 0.75 m. In particular, show all values at z = 0.1 m, 0.6 m, and 0.75 m. Assume that the datum is located at z = 0 and that the soil is fully saturated.

4) Under the conditions in 3), the downstream water flow rate is 14 cm³ per minute. Calculate the coefficient of permeability k.

5) Under the conditions in 3), plot the total stress, pore pressure, and effective stress diagrams for the range z = 0.1 m to z = 0.6 m, indicating all values at 0.1 m intervals. Assume that the void ratio is the same that you calculated in 1).

(2) When plotting a flow net to analyze flow of water through an isotropic soil (hydraulic conductivity is the same in all directions), the streamlines (flow channels) and equipotential lines form square shapes. Explain why (you may use a figure if necessary).

[Question 3] Figure 4 represents a soil with alternate deposit layers of sand and clay. The physical properties of each layer are shown within the figure. The clay in the clay layer is normally consolidated, and the water table lies at a depth of 2 m from the ground surface. Below the water table, all soils are fully saturated with water, and it is assumed that the pore water pressure is consistent with the hydrostatic pressure. Answer the following questions considering the sand layers as perfect drainage material and the unit weight of water as γ_w = 9.8 kN/m³.

1) Calculate the vertical total stress, pore water pressure, and vertical effective stress, at the center of the clay layer.

2) If a uniform surcharge load q = 50 kN/m² is applied on the ground surface, calculate the vertical total stress, pore water pressure, and vertical effective stress at the center of the clay layer after a sufficient amount of time has passed. At that time, calculate the total consolidation settlement of the clay layer, and its final void ratio. Assume that the stress in the center of the clay layer can be used as representative of the whole clay layer, and that changes in the physical properties of the soils associated with the consolidation settlement and depth are negligible.

3) Derive Terzaghi’s one-dimensional consolidation equation expressed by the following formula. Explain the assumptions and variables used in its formulation.

\[ \frac{\partial u}{\partial t} = \frac{k}{\gamma_w \partial z^2} \frac{\partial^2 u}{\partial z^2} \]

Here, u is the excess pore water pressure, t is time, k is the coefficient of soil permeability,
is the coefficient of volume compressibility, and \( z \) is the vertical coordinate.

(4) If the coefficient of soil permeability for the clay layer is \( 1.0 \times 10^{-9} \) m/s, and its coefficient of volume compressibility is \( 2.5 \times 10^{-4} \) m\(^2\)/kN, calculate the time required to reach 90% of the total consolidation (Time factor \( T_v = 0.848 \)).

![Figure 4 Soil with alternate deposits of sand and clay](image)

[Question 4] The stress state of a given point in the ground where the horizontal and vertical directions coincide with the directions of the principal stresses, is shown in Figure 5. Using cohesion \( c \) and internal friction angle \( \phi \), answer the following questions.

![Figure 5 Stress state of in the ground](image)

(1) When the principal stress \( \sigma_z = \sigma_a \) in the vertical direction and the principal stress \( \sigma_x = \sigma_b \) in the horizontal direction, the state of stress of the soil stays admissible (does not reach
failure condition). Draw the Mohr’s stress circle of this condition by taking $\sigma_a > \sigma_b$, as well as the Mohr-Coulomb failure criterion.

(2) From the state of (1), while keeping the principal stress in the horizontal direction $\sigma_x = \sigma_b$ constant, the principal stress in the vertical direction $\sigma_z$ is gradually increased from $\sigma_a$ until it eventually reaches failure when $\sigma_z = \sigma_c$. Draw the Mohr’s stress circle of this condition as well as the Mohr-Coulomb failure criterion.

(3) In regard to (2), using the geometrical relation between a Mohr’s stress circle and the Mohr-Coulomb failure criterion, formulate $\sigma_c$ in terms of $\sigma_b$, $c$ and $\phi$.

(4) According to the state of stress given in (2), determine the normal stress and shear stress that satisfy the failure criteria. In addition, find the angle of the failure plane inclined from the horizontal plane.